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Prospective Elementary Mathematics Teachers' Thought Processes on a Model Eliciting Activity

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Abstract

Mathematical model and modeling are one of the topics that have been intensively discussed in recent years. The purpose of this study is to examine prospective elementary mathematics teachers' thought processes on a model eliciting activity and reveal difficulties or blockages in the processes. The study includes forty-five seniors taking the course of Modeling in Teaching Mathematics in an elementary education program at a university. Three prospective teachers were selected among them and then interviewed in a focus group. The transcription of conversation of the group was examined and qualitatively analyzed. Findings indicated that prospective teachers were able to successfully work with modeling eliciting activity and improve their mathematical understandings. They also showed some difficulties while working on the modeling activity.

Key Words

Model Eliciting Activity, Prospective Teachers, Elementary Education, Mathematics Modeling.

Top of Form

In recent years, research studies in mathematics education have been increasingly interested in mathematical model and modeling because of the need to establish the relationships between the real world and mathematics (Lesh, Hamilton, & Kaput, 2007). Many questions and problems about individual learning and teaching of mathematics have affected the relationship of mathematics to the real world (Blum, Galbraith, Henn, & Niss, 2002). PISA (Program for International Student Assessment) studies focusing on individual's ability to relate mathematics to the real world have particularly encouraged this type of study (OECD, 1999). In line with the results of the PISA studies, researchers in many countries have begun to question how much students in school-education system are prepared to solve the real-world problems they encounter in their future professional lives (Blum, 2002; English, 2006; Mousoulides, 2007). As a result, mathematics educators such as English (2002), Gainsbourg (2006) and Lesh and Doerr (2003) have begun to emphasize the importance of new skills and understanding for success in beyond the school. These are: (1) constructing, hypothesizing, describing, manipulating, predicting and understanding complex systems, (2) planning and working for complex and multifaceted problems that require critical communication skill and (3) adapting to work on conceptual systems developing continuously. When students increasingly face this

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kind of situations in their daily life, it is important to make sure that students have enough experience to work together and interpret mathematical situations that enable them to think in different ways and share their ideas with their peers. Thus, model eliciting activities are one of main tools that help students to gain experiences and the new skills required (Blum & Niss, 1991; English & Watters, 2005; Lesh & Doerr, 2003).

Research studies in elementary education level showed that model eliciting activities (*a*) are a powerful tool that helps students to develop critical and higher level thinking skills (English & Watters, 2005), (*b*) provide a new and effective learning environment in which students reveal and rebuild their existing conceptual knowledge (Chamberlin, 2004), (*c*) encourage the use of different and multiple representations to explain mathematical structure and conceptual systems (Boaler, 2001; English & Watters, 2004; Mousoulides, 2007) and (*d*) improve students' communication skills in sharing of their understanding of mathematical ideas (English, 2006). On the other hand, students had difficulties in the following modeling processes: understanding the problem, structuring and simplifying the problem, developing appropriate assumptions, exploring relationships between variables, questioning the relationship between the model and real-life and validating the model obtained (Blum & Leib, 2007; Crouch & Haines, 2007; Maab, 2007; Sol, Gimenez, & Rosich, 2011). It was argued that these processes were affected by students' mathematical thinking styles, their own life and problem-related experiences, their beliefs and attitudes about mathematics and model eliciting activities (Ferri, 2011; Schoenfeld, 1992).

Many countries such as United States, Britain, Australia, the Netherlands, Germany, and Sweden carried out important projects on model eliciting activities to adapt in their mathematics programs (Blum & Niss, 1991; Mousoulides, Sriraman, & Christou, 2007). Similarly the Turkish government put into practice a new mathematics education program particularly focusing on mathematical modeling and higher level mathematical thinking. The vision of the new program is to help students to develop analytical thinking and reasoning skills, establish the relationship between mathematics and real life situations and create different solutions to the problems they face in their everyday life (Milli Eğitim Bakanlığı [MEB], 2005). Developing such skills depends on the development of modeling skills allowing students to create their own mathematical ideas and understanding and reach general, valid and usable solutions (English, 2006). At this point, the question raised is whether prospective teachers who would teach mathematics modeling to their students have enough mathematical knowledge and skills needed. Therefore, this study aims to examine prospective elementary mathematics teachers' thought processes on a model eliciting activity and reveal difficulties or blockages in the processes.

Method

Research Model

This is a qualitative research study that aims to examine prospective elementary mathematics teachers' thought processes on a model eliciting activity and also reveal difficulties or blockages in this process. The case study design, which is defined to examine or analyze in depth of a case or a group, was selected for research design. The case in this study was the focus group of prospective teachers who were working on the model eliciting activity.

Participants

This research study was carried on a university located in the Black Sea region in the 2009-2010 fall semester. The participants were forty-five senior students who were taking the course of *Modeling in Mathematics Teaching* in the department of mathematics education. In a period of fourteen-week course, students worked individually or as a group on different model eliciting activities that require mathematical modeling. In this processes, students also discussed the issues of model, modeling, mathematical modeling, model eliciting activity, problem solving and differences among them. At the end of the semester, three students (one girl and two boys) were selected as a focus group using criterion sampling on the basis of the answers given to modeling problems. Some other criteria such as being successful, self-confident, talkative, articulate and previously worked with each other were also being considered in the selection process.

Data Collection Instrument

At the end of the semester, the Team Ranking Problem (Appendix) was given to the focus group to work on it in a classroom environment. The Team Ranking Problem is a model eliciting activity

that has many solution paths and end-points (Lesh, Hoover, Hole, Kelly, & Post, 2000). Unlike the traditional mathematical problems, model eliciting activities are non-routine tasks that ask students to mathematically interpret a complex real-world situation and require them to formulate a mathematical description, procedure or method for the purpose of making a decision (Lesh & Zawojewsky, 2007; Mousoulides, 2007). At the end of the process, the group's task was to use the win-loss record to develop a model for ranking the top five teams. A total of 90 minutes interview with the group was video-recorded and students' written solutions were also collected at the end.

Data Analysis

While working on the Team Ranking Problem, prospective teachers' thoughts and written solutions were analyzed through the lens of Lester and Kehle's (2003) *ideal model of mathematical activity*. In the process, models developed by the students and elements that make up the model are considered. To increase the transferability of the research, research procedure, research design, participants, data collection instrument and processes, and analysis were explained in detail. In addition, two other faculty members who have experience on qualitative research checked the modeling processes and had full agreement on the interpretations of the direct quotations used.

Theoretical Framework

Mathematical modeling is an effective tool that helps students to prepare to solve real-life problems and allow them to use flexible and creative thinking (English, 2006; Lesh & Doerr, 2003). Mousoulides (2007) claims that modeling activities can be used as a way to develop critical thinking and *mathematical literacy* by referring to NCTM (2000) (National Council of Teachers of Mathematics)-report that recommends the use of purposeful activities and questioning techniques to improve understanding of the relationships between mathematical concepts. Research studies revealed that students who deal with modeling activities were able to successfully work on complex, multifaceted tasks and develop the existing understanding of mathematical concepts (English, 2006). Modeling activities help students to use many different solution paths and interpretations and also develop the intrinsic motivation (Mousoulides et al., 2010). In addition, students engage in high-level mathematical thinking processes such as reasoning, constructing, analyzing and describing while mathematizing relationships, patterns or rules (Lesh & Doerr). Therefore, as opposed to the traditional approach to teaching mathematics, modeling activities provide rich learning opportunities that help students to rebuild their previous understandings and encourage in-depth thinking to find generalizable solutions (English, 2003, 2006).

Lester and Kehle (2003) expanded the approach of problem solving to a much broader concept of *mathematical activity* and gave an important role to metacognitive actions engaged in by the individual. Thus, they developed and explained a four-stage modeling process and called it as *ideal model of mathematical activity.* The new model is explained by the following: (1*) simplifying /problem posing*: a realistic and mathematical context poses a specific problem situation. To begin to solve the problem, the individual simplifies the complex setting by identifying the concepts and process related to the problem, (2) *abstracting:* this includes the selection of mathematical concepts and notations to represent the essential characteristics of the realistic model, (3) *computing:* this process involves manipulating expressions and deducing some mathematical conclusions and (4) *interpretation:* the final process involves the individual in comparing the results or solutions with original context and problem.

Findings

Prospective teachers carried out three main cycles while working on the Team Ranking Problem. First, they simply ranked the teams where they were in the graph. In the second stage, as happened to the work of Boaler (2001), English and Watters (2004) and Mousoulides (2007), who found that students developed a complex and multi-tier model and successfully used mathematical concepts such as calculating, sorting, making table and analyzing relationships. Finally, they end up with a simple model by reducing the number of variables of the model obtained.

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In the first stage, students decided that the most wins took place "highest" on the graph while the most losses took place "farthest to the right" on the graph. In this process, group members used a limited mathematical thinking by only taking

into account wins and losses axes. In the second stage, students numbered the axes and then made assumptions that each team played twice with the other team on the possible results of a win, loss, or tie. By taking into consideration the real football leagues, they made a table listing a total number of games played, the numbers of wins, losses and ties for each team. They then developed a model of "scoring system" in assigning different points to the win, loss and tie, and then calculated and listed the total scores of each team. When two or more teams had equal score, students changed the scoring system to resolve the ties, but each time they end up with the same situation. As a result, group members failed to develop alternative methods when their models were not effective for ranking teams. In the final stage, in order to reach a conclusion, prospective teachers turned back to the starting point of the modeling process and compared the teams two by two and completed the activity with a simpler model.

Discussion and Conclusions

This study revealed that prospective teachers were able to successfully work on the model eliciting activities and develop their own existing mathematical understanding and improve their communication skills while having difficulty in some stages of modeling processes. Prospective teachers in all of the process worked continuously on the modeling activity in a non-linear manner from the beginning to the end and engaged in many cognitive and meta-cognitive thinking processes (English & Watters, 2005). They tested various assumptions, compared results in real situations whether or not appropriate and had many arguments discussed until they reached the conclusion (English, 2006). In other words, the Team Ranking Problem provided a new learning environment for students in giving an opportunity to discover, deeply think, research and develop their own mathematical ideas (Chamberlin, 2004). The difference between modeling activities and traditional problems is that givens in the modeling activities are not precise and clear. Due to lack of these data, students had difficulties to determine variables used in the solution. This result is consistent with the work of Blum and Leib (2007) and Crouch and Haines (2007), who found that students had difficulty in the first step of modeling processes. After that, group members identified the wins, losses, and ties and then calculated each team's score. However, every time they came up with the same score for the two teams. They then used a tie-breaking strategy and changed the points they assigned for the wins, losses and ties, and re-calculated the total scores of the teams listed. But, the new model was not able to solve this problem as happened to the work of Maab (2007), who found that students were not able to develop adequate and effective mathematical models to the real problem. As a result, the students returned to the very beginning of modeling process and they ranked the teams only based on the wins and losses by taking out the tie-situation. It is clearly showed that prospective teachers faced difficulties in the transition process of real world problem to mathematical model. This result is also supported by the work of Sol et al. (2011), who found that students had obstacles in developing alternative models in line with the actual situation. To overcome this difficulty, group members reduced the number of variables and used a simpler, non-systematic and more limited model by only taking into account the wins and losses. The reason could be the teacher-centered system showing students what and how to do by only focusing on the result rather than the solution processes. Students' lack of experience of working together, generating new ideas, developing and testing assumptions that require modeling problems could be additional effect to this reason. By gaining experiences on modeling activities, Blum and Ferri (2009) recommend a balance between maximal independence of students and minimal guidance by teachers.

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